

Deep Learning with Keras

ACTL3143 & ACTL5111 Deep Learning for Actuaries
Patrick Laub



Lecture Outline

- **California House Price Prediction**
- EDA & Baseline Model
- Our First Neural Network
- Force positive predictions
- Preprocessing
- Early Stopping
- Quiz



Import the data

```

1 from sklearn.datasets import fetch_california_housing
2
3 features, target = fetch_california_housing(
4     as_frame=True, return_X_y=True)
5 features

```

	MedInc	HouseAge	AveRooms	AveBedrms	Population	AveOccup	Latitude	Lon
0	8.3252	41.0	6.984127	1.023810	322.0	2.555556	37.88	-122
1	8.3014	21.0	6.238137	0.971880	2401.0	2.109842	37.86	-122
2	7.2574	52.0	8.288136	1.073446	496.0	2.802260	37.85	-122
...
20637	1.7000	17.0	5.205543	1.120092	1007.0	2.325635	39.43	-121.
20638	1.8672	18.0	5.329513	1.171920	741.0	2.123209	39.43	-121.
20639	2.3886	16.0	5.254717	1.162264	1387.0	2.616981	39.37	-121.

20640 rows × 8 columns



What is the target?

```
1 target
```

```
0      4.526  
1      3.585  
2      3.521
```

```
...  
20637  0.923  
20638  0.847  
20639  0.894
```

```
Name: MedHouseVal, Length: 20640, dtype: float64
```



The dataset

The target variable is the median house value for California districts, expressed in hundreds of thousands of dollars (\$100,000).

This dataset was derived from the 1990 U.S. census, using one row per census block group. A block group is the smallest geographical unit for which the U.S. Census Bureau publishes sample data (a block group typically has a population of 600 to 3,000 people).



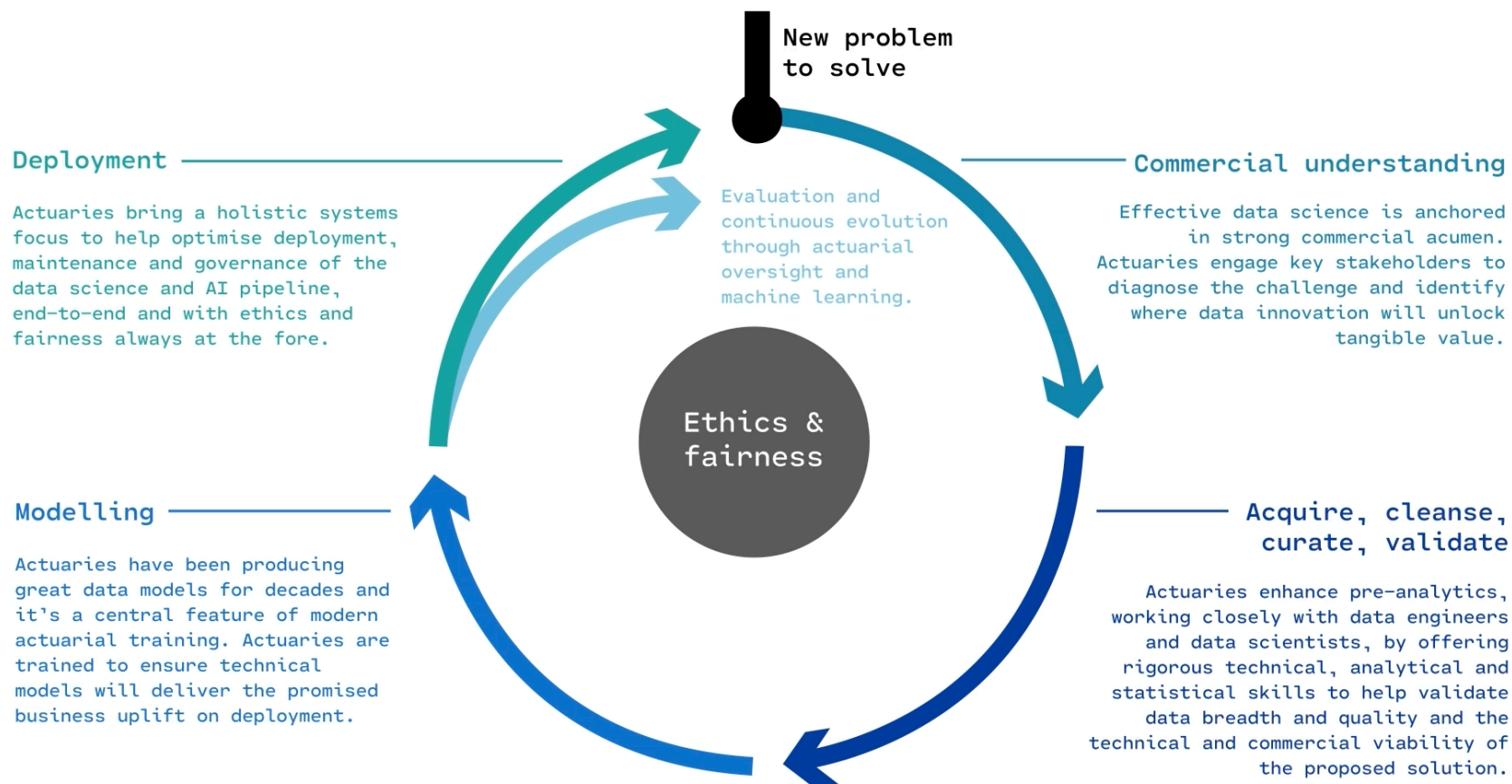
Columns

- `MedInc` median income in block group
- `HouseAge` median house age in block group
- `AveRooms` average number of rooms per household
- `AveBedrms` average # of bedrooms per household
- `Population` block group population
- `AveOccup` average number of household members
- `Latitude` block group latitude
- `Longitude` block group longitude

Source: [Scikit-learn documentation](#).



An entire ML project



ML life cycle

Source: Actuaries Institute, [Do Data Better](#).



Questions to answer in ML project

You fit a few models to the training set, then ask:

1. **(Selection)** Which of these models is the best?
2. **(Future Performance)** How good should we expect the final model to be on unseen data?



Set aside a fraction for a test set

```

1 from sklearn.model_selection import train_test_split
2
3 X_train, X_test, y_train, y_test = train_test_split(
4     features, target, random_state=42
5 )

```

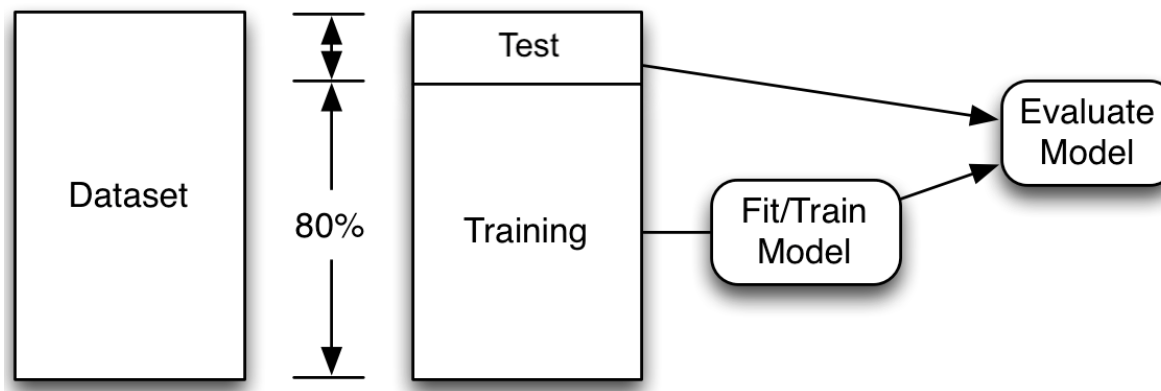


Illustration of a typical training/test split.

Note: Compare X_{train} / y_{train} names, capitals & lowercase.

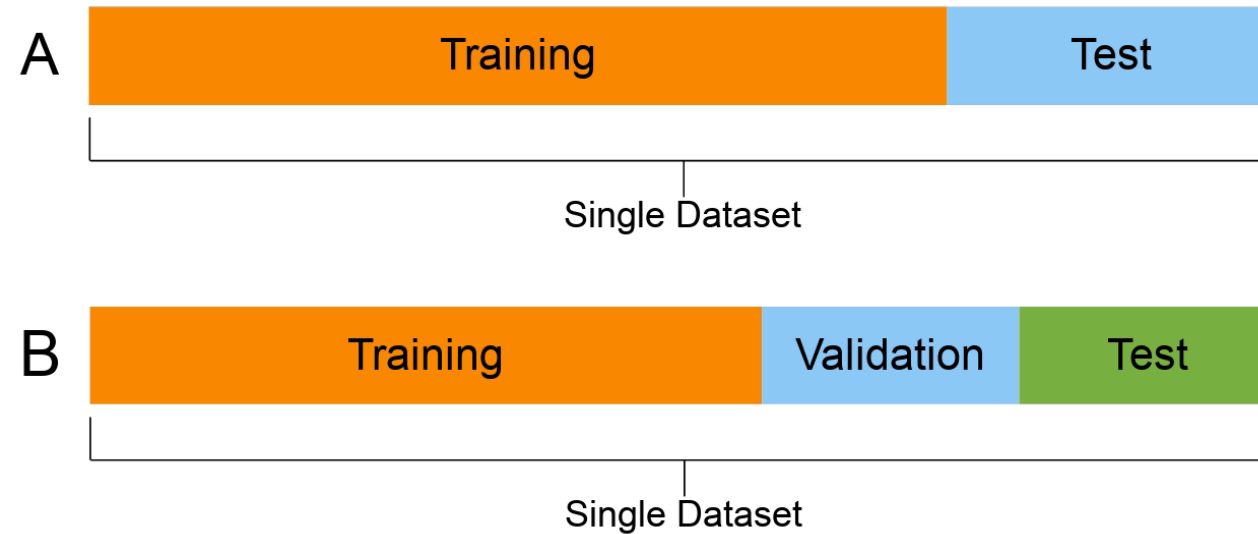


Our use of sklearn.



Adapted from: Heaton (2022), [Applications of Deep Learning](#), Part 2.1: Introduction to Pandas, and [this random site](#).

Basic ML workflow



Splitting the data.

1. For each model, fit it to the *training set*.
2. Compute the error for each model on the *validation set*.
3. Select the model with the lowest validation error.
4. Compute the error of the final model on the *test set*.

Source: [Wikipedia](#).



Split three ways

```
1 # Thanks https://datascience.stackexchange.com/a/15136
2 X_main, X_test, y_main, y_test = train_test_split(
3     features, target, test_size=0.2, random_state=1
4 )
5
6 # As 0.25 x 0.8 = 0.2
7 X_train, X_val, y_train, y_val = train_test_split(
8     X_main, y_main, test_size=0.25, random_state=1
9 )
10
11 X_train.shape, X_val.shape, X_test.shape
```

((12384, 8), (4128, 8), (4128, 8))



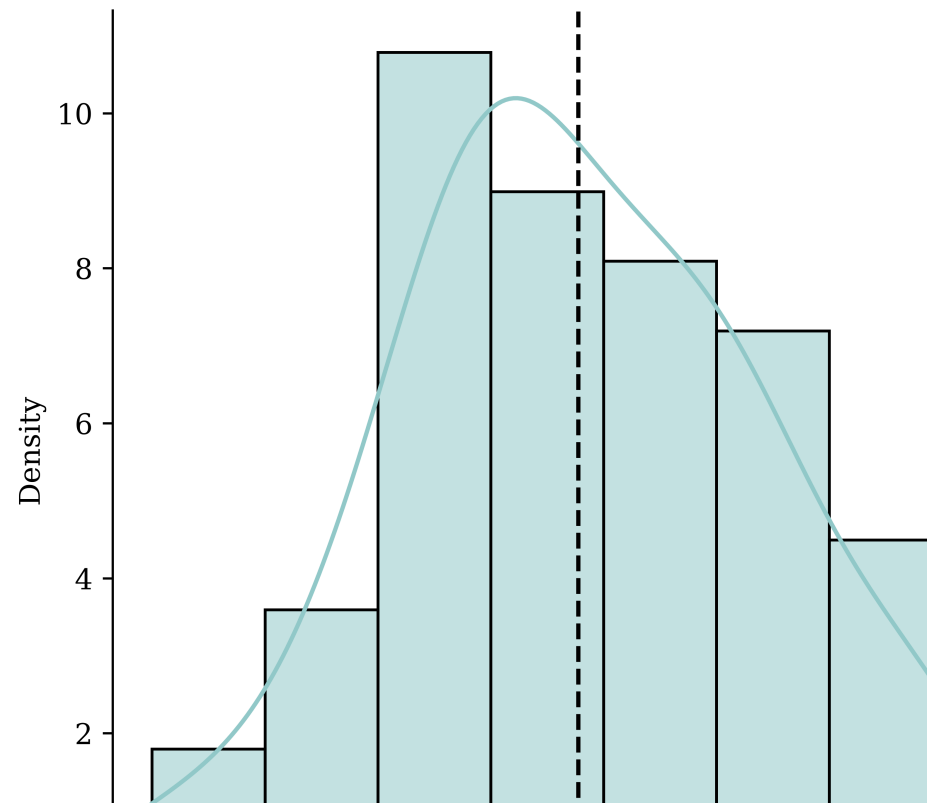
Why not use test set for both?

Thought experiment: have m classifiers: $f_1(\mathbf{x}), \dots, f_m(\mathbf{x})$.

They are just as good as each other in the long run

$$\mathbb{P}(f_i(\mathbf{X}) = Y) = 90\%, \quad \text{for } i = 1, \dots, m.$$

Evaluate each model on the test set, some will be better than others.



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The training set

```
1 X_train
```

	MedInc	HouseAge	AveRooms	AveBedrms	Population	AveOccup	Latitude	Longitude
9107	4.1573	19.0	6.162630	1.048443	1677.0	2.901384	34.63	-118.0
13999	0.4999	10.0	6.740000	2.040000	108.0	2.160000	34.69	-116.0
5610	2.0458	27.0	3.619048	1.062771	1723.0	3.729437	33.78	-118.0
...
8539	4.0727	18.0	3.957845	1.079625	2276.0	2.665105	33.90	-118.0
2155	2.3190	41.0	5.366265	1.113253	1129.0	2.720482	36.78	-119.0
13351	5.5632	9.0	7.241087	0.996604	2280.0	3.870968	34.02	-117.0

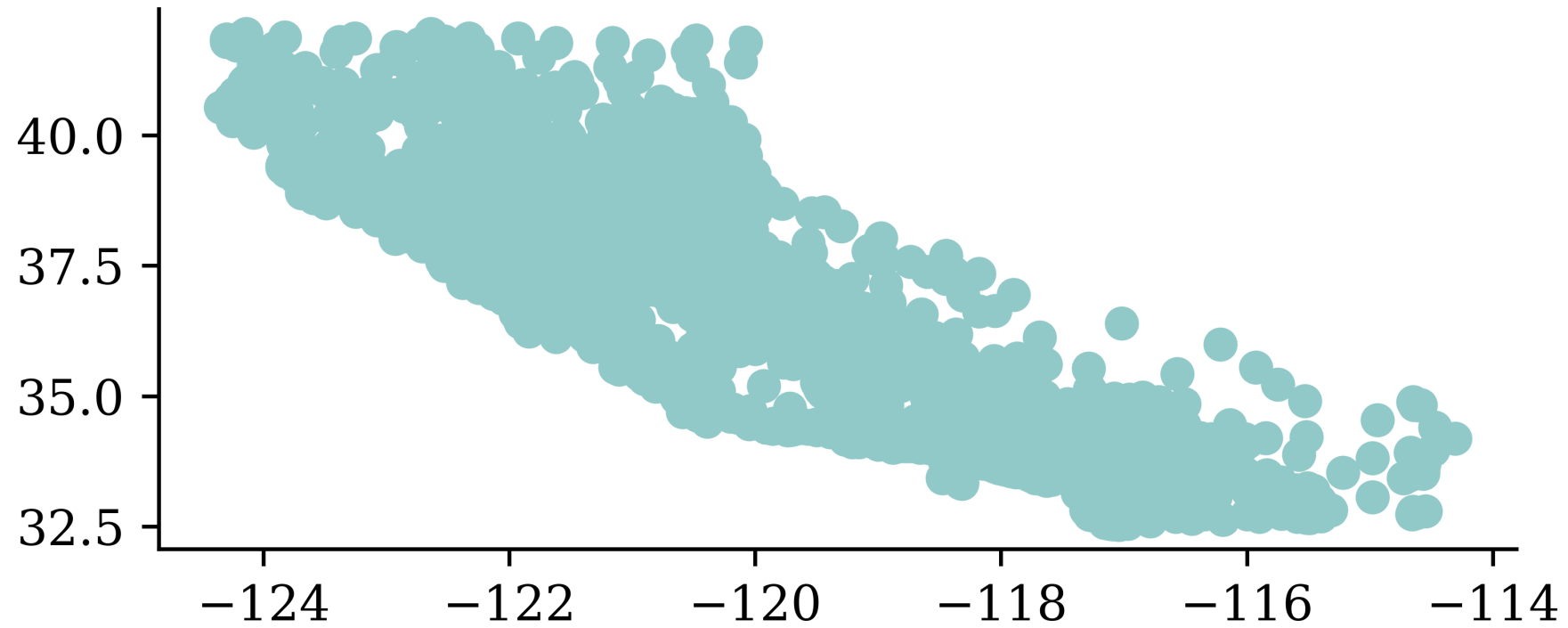
12384 rows × 8 columns



Location

Python's `matplotlib` package \approx R's basic `plots`.

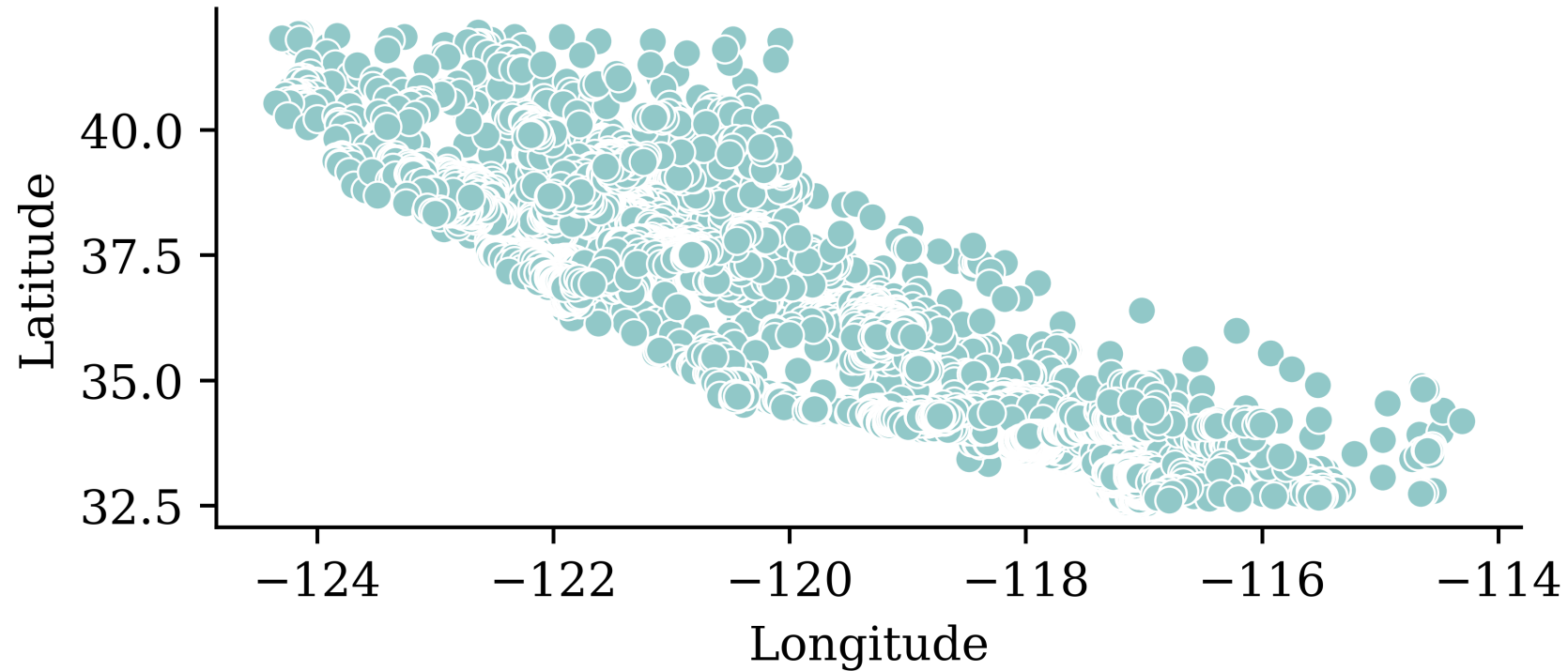
```
1 import matplotlib.pyplot as plt
2
3 plt.scatter(features["Longitude"], features["Latitude"])
```



Location #2

Python's `seaborn` package \approx R's `ggplot2`.

```
1 import seaborn as sns
2
3 sns.scatterplot(x="Longitude", y="Latitude", data=features);
```



Features

```
1 print(list(features.columns))
```

```
['MedInc', 'HouseAge', 'AveRooms', 'AveBedrms', 'Population', 'AveOccup', 'Latitude',  
'Longitude']
```

How many?

```
1 num_features = len(features.columns)  
2 num_features
```

8

Or

```
1 num_features = features.shape[1]  
2 features.shape
```

(20640, 8)



Linear Regression

$$\hat{y} = w_0 + \sum_{i=1}^N w_i x_i.$$

```
1 from sklearn.linear_model import LinearRegression
2
3 lr = LinearRegression()
4 lr.fit(X_train, y_train);
```

The w_0 is in `lr.intercept_` and the others are in

```
1 print(lr.coef_)

[ 4.34267965e-01  9.88284781e-03 -9.39592954e-02  5.86373944e-01
 -1.58360948e-06 -3.59968968e-03 -4.26013498e-01 -4.41779336e-01]
```



Make some predictions

```
1 X_train.head(3)
```

	MedInc	HouseAge	AveRooms	AveBedrms	Population
9107	4.1573	19.0	6.162630	1.048443	1677.0
13999	0.4999	10.0	6.740000	2.040000	108.0
5610	2.0458	27.0	3.619048	1.062771	1723.0

```
1 y_pred = lr.predict(X_train.head(3))
2 y_pred
```

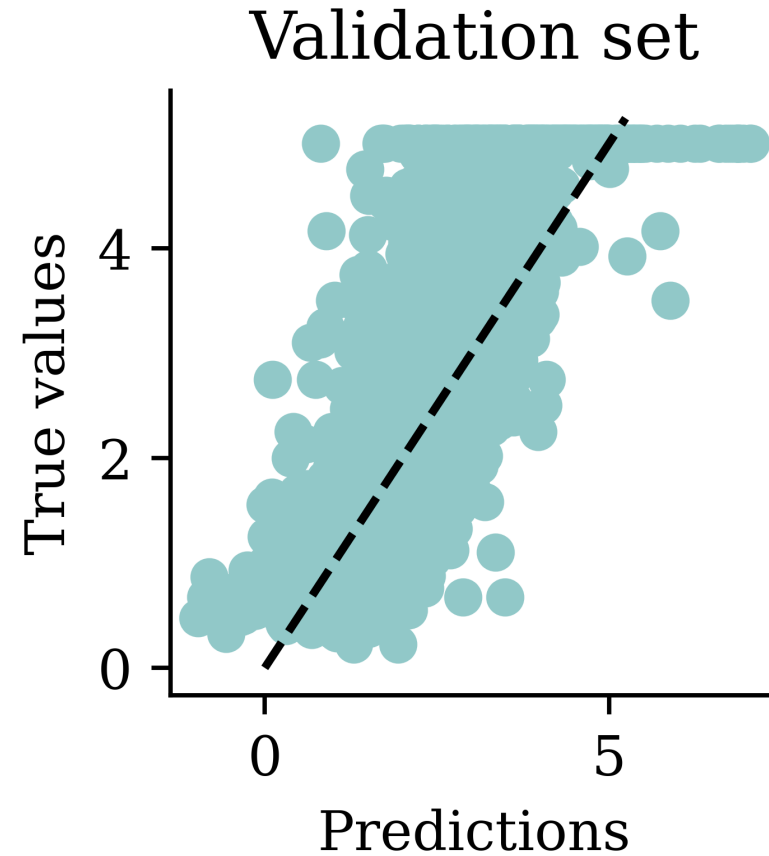
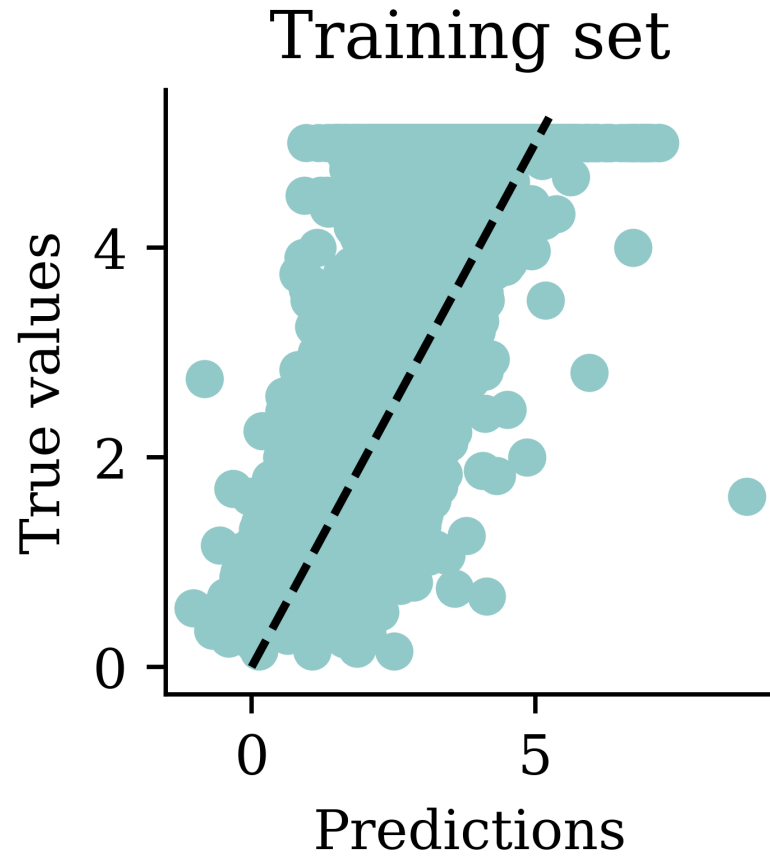
```
array([1.81699287, 0.0810446 , 1.62089363])
```

```
1 prediction = lr.intercept_
2 for w_i, x_i in zip(lr.coef_, X_train.iloc[0]):
3     prediction += w_i * x_i
4 prediction
```

```
1.8169928680677856
```



Plot the predictions



Calculate mean squared error

```

1 import pandas as pd
2
3 y_pred = lr.predict(X_train)
4 df = pd.DataFrame({"Predictions": y_pred, "True values": y_train})
5 df["Squared Error"] = (df["Predictions"] - df["True values"]) ** 2
6 df.head(4)

```

	Predictions	True values	Squared Error
9107	1.816993	2.281	0.215303
13999	0.081045	0.550	0.219919
5610	1.620894	1.745	0.015402
13533	1.168949	1.199	0.000903

```
1 df["Squared Error"].mean()
```

0.5291948207479794



Using `mean_squared_error`

```
1 df["Squared Error"].mean()
```

```
0.5291948207479794
```

```
1 from sklearn.metrics import mean_squared_error as mse
2
3 mse(y_train, y_pred)
```

```
0.5291948207479794
```

Store the results in a dictionary:

```
1 mse_lr_train = mse(y_train, lr.predict(X_train))
2 mse_lr_val = mse(y_val, lr.predict(X_val))
3
4 mse_train = {"Linear Regression": mse_lr_train}
5 mse_val = {"Linear Regression": mse_lr_val}
```



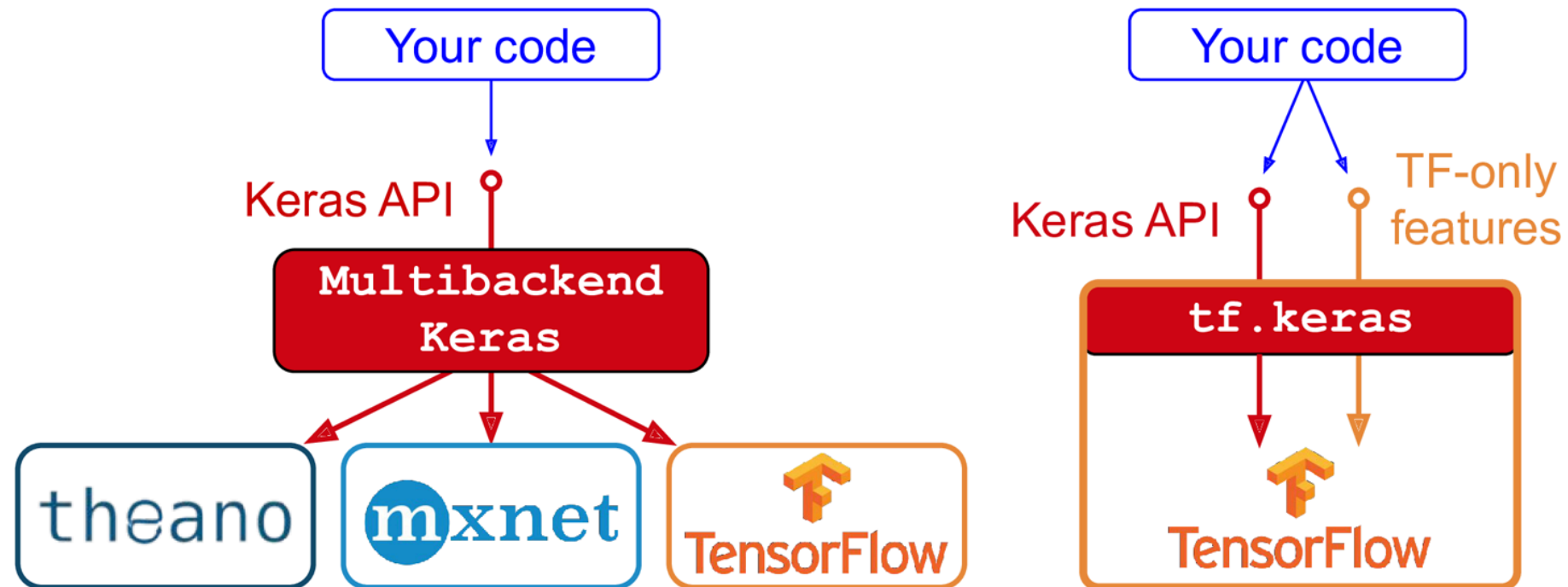
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What are Keras and TensorFlow?

Keras is common way of specifying, training, and using neural networks. It gives a simple interface to *various backend* libraries, including Tensorflow.



Keras as a independent interface, and Keras as part of Tensorflow.



Source: Aurélien Géron (2019), *Hands-On Machine Learning with Scikit-Learn, Keras, and TensorFlow*, 2nd Edition, Figure 10-10.

Create a Keras ANN model

Decide on the architecture: a simple fully-connected network with one hidden layer with 30 neurons.

Create the model:

```
1 from keras.models import Sequential
2 from keras.layers import Dense, Input
3
4 model = Sequential(
5     [Input((num_features,)),
6     Dense(30, activation="relu"),
7     Dense(1)]
8 )
```



Inspect the model

```
1 model.summary()
```

Model: "sequential"

Layer (type)	Output Shape	Param #
dense (Dense)	(None, 30)	270
dense_1 (Dense)	(None, 1)	31

Total params: 301 (1.18 KB)

Trainable params: 301 (1.18 KB)

Non-trainable params: 0 (0.00 B)



The model is initialised randomly

```
1 model = Sequential([Dense(30, activation="relu"), Dense(1)])  
2 model.predict(X_val.head(3), verbose=0)
```

```
array([[ -737.1918  ],  
       [-446.86752 ],  
       [ -20.181843]], dtype=float32)
```

```
1 model = Sequential([Dense(30, activation="relu"), Dense(1)])  
2 model.predict(X_val.head(3), verbose=0)
```

```
array([[ -566.09283 ],  
       [-336.90277 ],  
       [ -35.532364]], dtype=float32)
```



Controlling the randomness

```
1 import random
2
3 random.seed(123)
4
5 model = Sequential([Dense(30, activation="relu"), Dense(1)])
6
7 display(model.predict(X_val.head(3), verbose=0))
8
9 random.seed(123)
10 model = Sequential([Dense(30, activation="relu"), Dense(1)])
11
12 display(model.predict(X_val.head(3), verbose=0))
```

```
array([[ -513.13403 ],
       [-302.18195 ],
       [ -17.794558]], dtype=float32)
```

```
array([[ -513.13403 ],
       [-302.18195 ],
       [ -17.794558]], dtype=float32)
```



Fit the model

```
1 random.seed(123)
2
3 model = Sequential([
4     Dense(30, activation="relu"),
5     Dense(1)
6 ])
7
8 model.compile("adam", "mse")
9 %time hist = model.fit(X_train, y_train, epochs=5, verbose=False)
10 hist.history["loss"]
```

CPU times: user 1.47 s, sys: 1.14 s, total: 2.61 s
Wall time: 1.38 s

```
[1294.4464111328125,
 2.6999971866607666,
 1.6647613048553467,
 1.4132919311523438,
 1.0312913656234741]
```



Make predictions

```
1 y_pred = model.predict(X_train[:3], verbose=0)
2 y_pred
```

```
array([[ 1.8309405],
       [-0.3535745],
       [ 1.0658419]], dtype=float32)
```

Note

The `.predict` gives us a ‘matrix’ not a ‘vector’. Calling `.flatten()` will convert it to a ‘vector’.

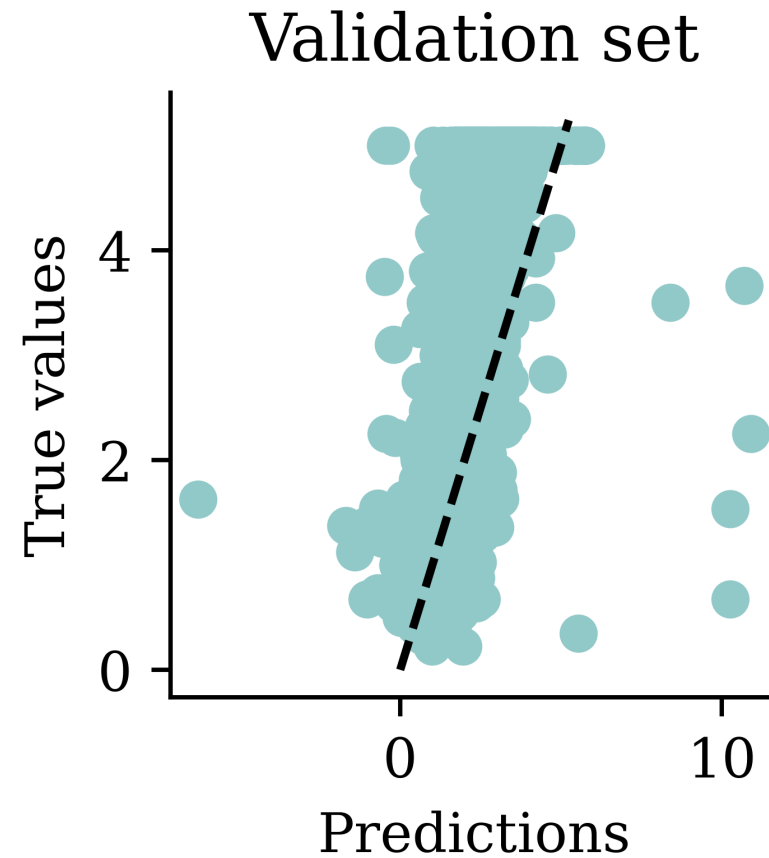
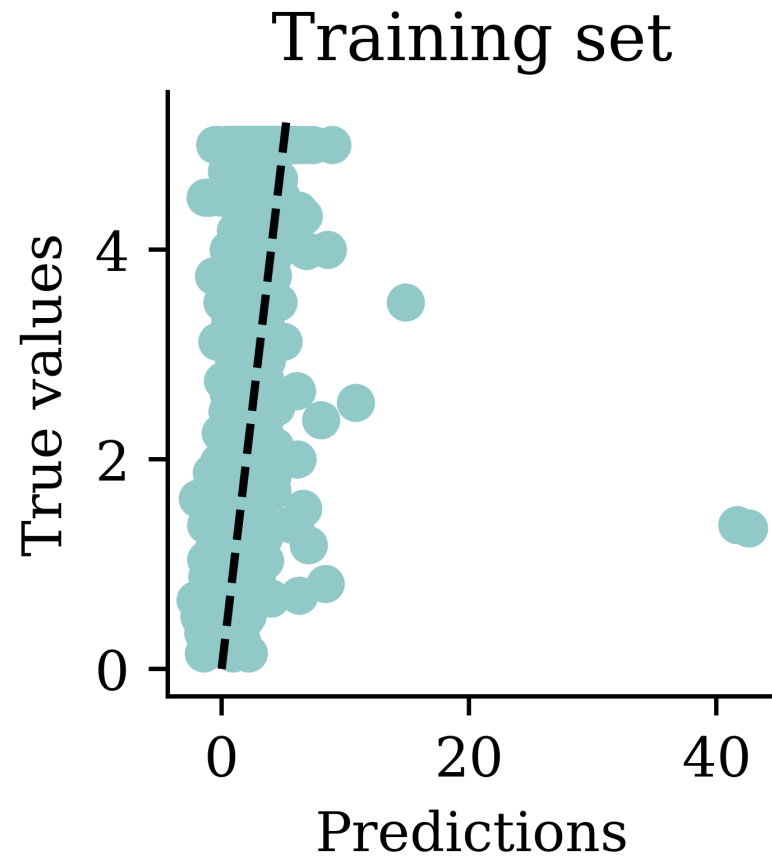
```
1 print(f"Original shape: {y_pred.shape}")
2 y_pred = y_pred.flatten()
3 print(f"Flattened shape: {y_pred.shape}")
4 y_pred
```

```
Original shape: (3, 1)
Flattened shape: (3,)
```

```
array([ 1.8309405, -0.3535745,  1.0658419], dtype=float32)
```



Plot the predictions



Assess the model

```
1 y_pred = model.predict(X_val, verbose=0)
2 mse(y_val, y_pred)
```

0.8456196916319004

```
1 mse_train["Basic ANN"] = mse(
2     y_train, model.predict(X_train, verbose=0)
3 )
4 mse_val["Basic ANN"] = mse(y_val, model.predict(X_val, verbose=0))
```

Some predictions are negative:

```
1 y_pred = model.predict(X_val, verbose=0)
2 y_pred.min(), y_pred.max()
```

(-6.2801437, 10.888435)

```
1 y_val.min(), y_val.max()
```

(0.225, 5.00001)



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Try running for longer

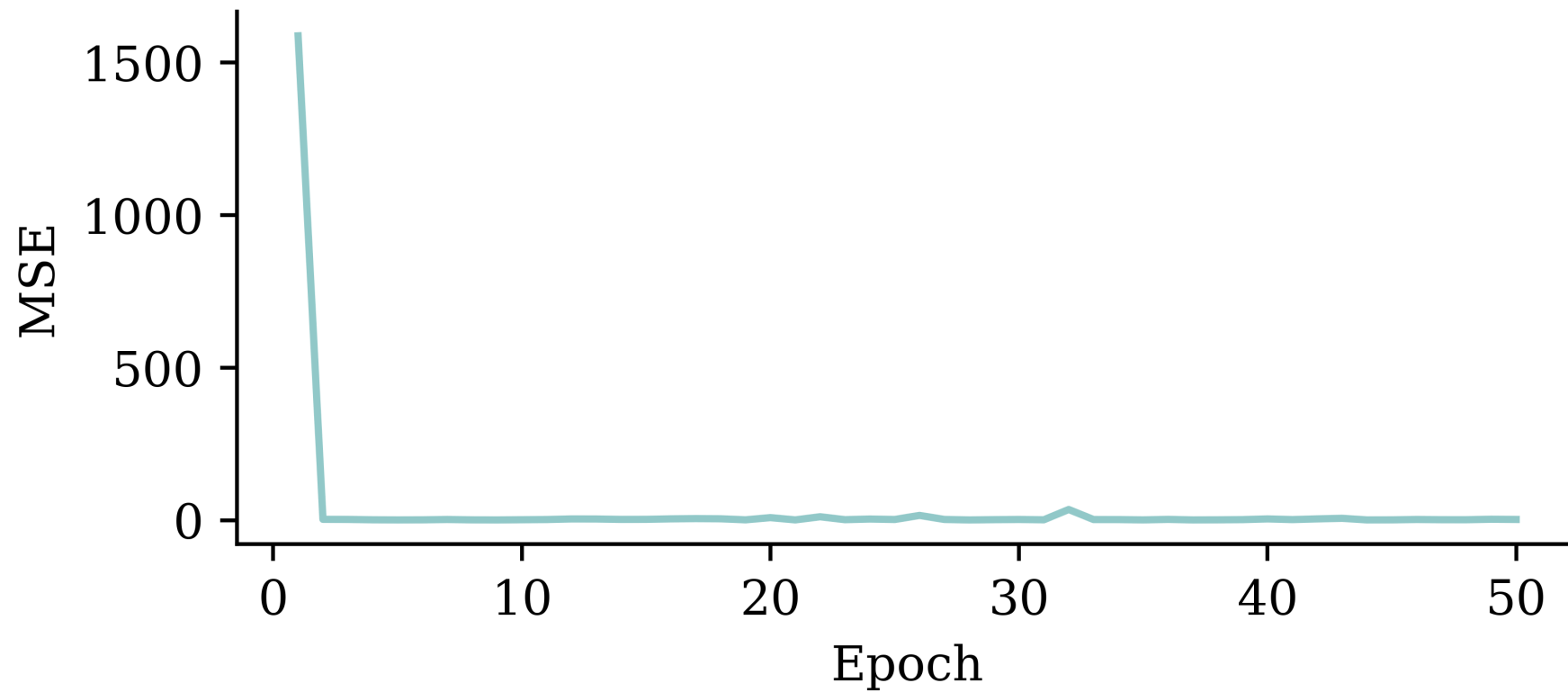
```
1 random.seed(123)
2
3 model = Sequential([
4     Dense(30, activation="relu"),
5     Dense(1)
6 ])
7
8 model.compile("adam", "mse")
9
10 %time hist = model.fit(X_train, y_train, \
11     epochs=50, verbose=False)
```

CPU times: user 13.5 s, sys: 8.32 ms, total: 13.5 s
Wall time: 13.6 s



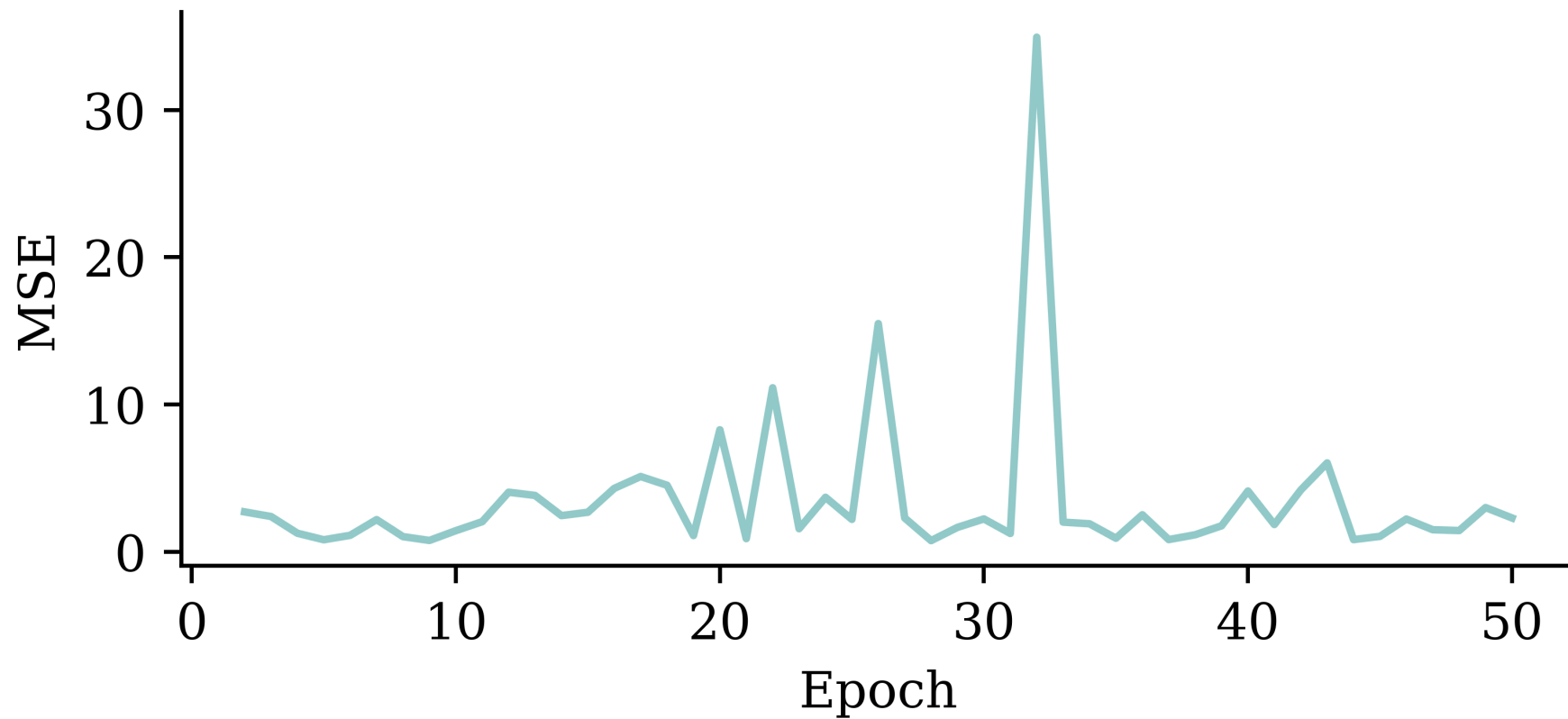
Loss curve

```
1 plt.plot(range(1, 51), hist.history["loss"])
2 plt.xlabel("Epoch")
3 plt.ylabel("MSE");
```



Loss curve

```
1 plt.plot(range(2, 51), hist.history["loss"][1:])  
2 plt.xlabel("Epoch")  
3 plt.ylabel("MSE");
```



Predictions

```

1 y_pred = model.predict(X_val, verbose=0)
2 print(f"Min prediction: {y_pred.min():.2f}")
3 print(f"Max prediction: {y_pred.max():.2f}")

```

Min prediction: -37.29
Max prediction: 7.75

```

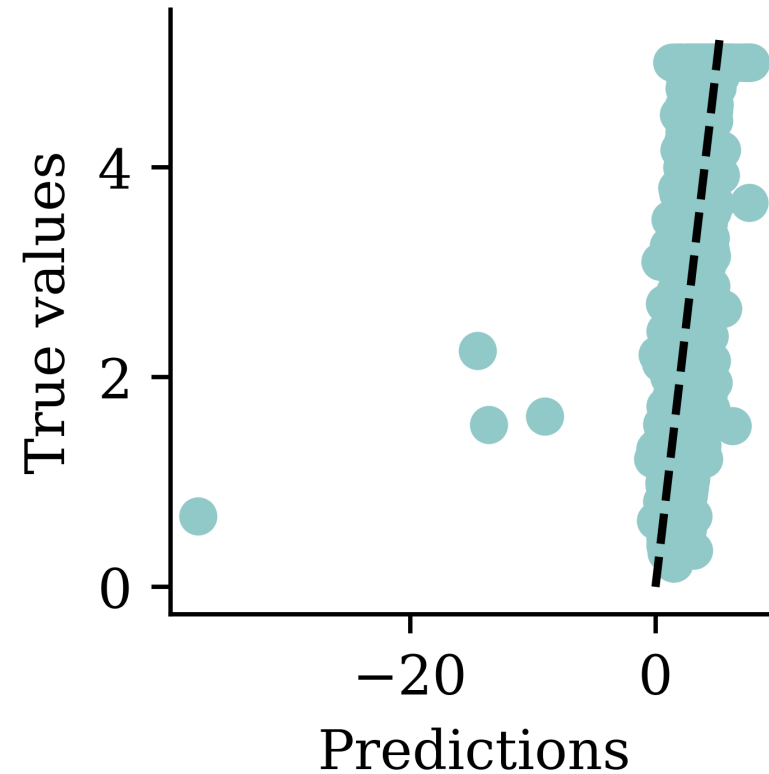
1 plt.scatter(y_pred, y_val)
2 plt.xlabel("Predictions")
3 plt.ylabel("True values")
4 add_diagonal_line()

```

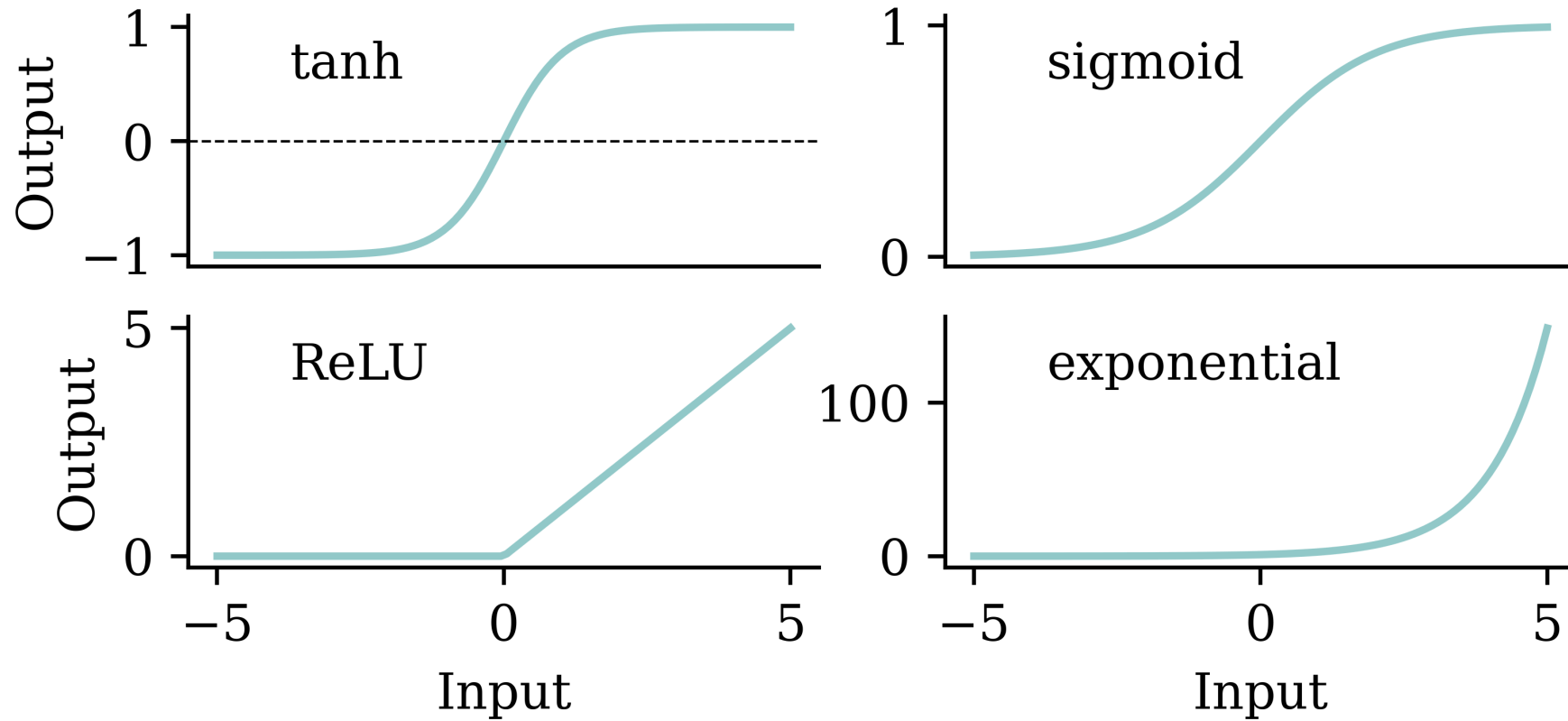
```

1 mse_train["Long run ANN"] = mse(
2     y_train, model.predict(X_train, ver
3 )
4 mse_val["Long run ANN"] = mse(y_val, mo

```



Try different activation functions



Enforce positive outputs (ReLU)

```
1 random.seed(123)
2
3 model = Sequential([
4     Dense(30, activation="relu"),
5     Dense(1, activation="relu")
6 ])
7
8 model.compile("adam", "mse")
9
10 %time hist = model.fit(X_train, y_train, epochs=50, \
11     verbose=False)
12
13 import numpy as np
14 losses = np.round(hist.history["loss"], 2)
15 print(losses[:5], " ... ", losses[-5:])
```

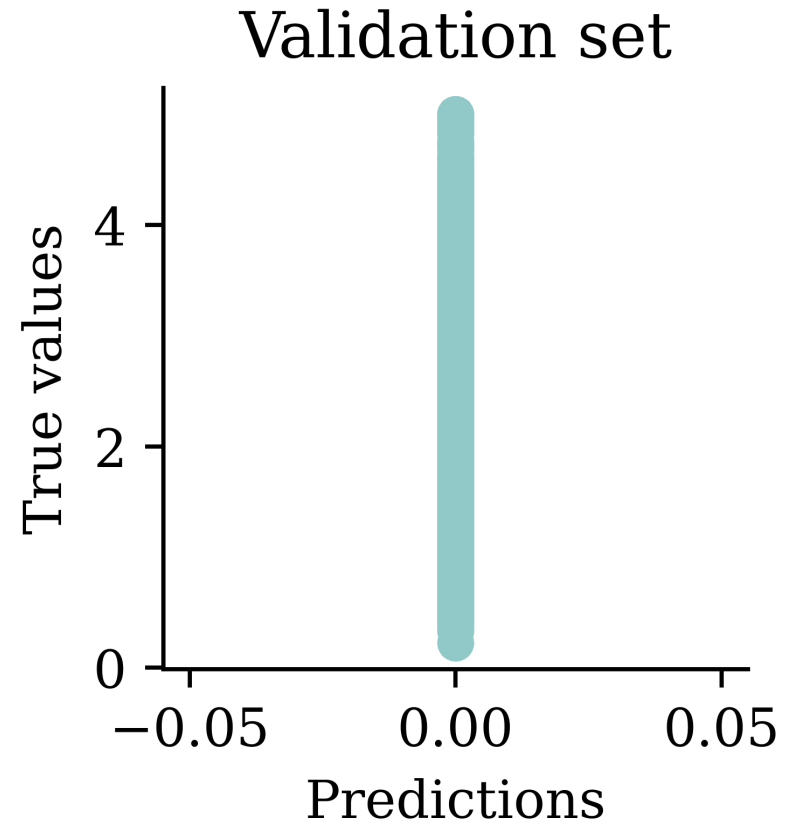
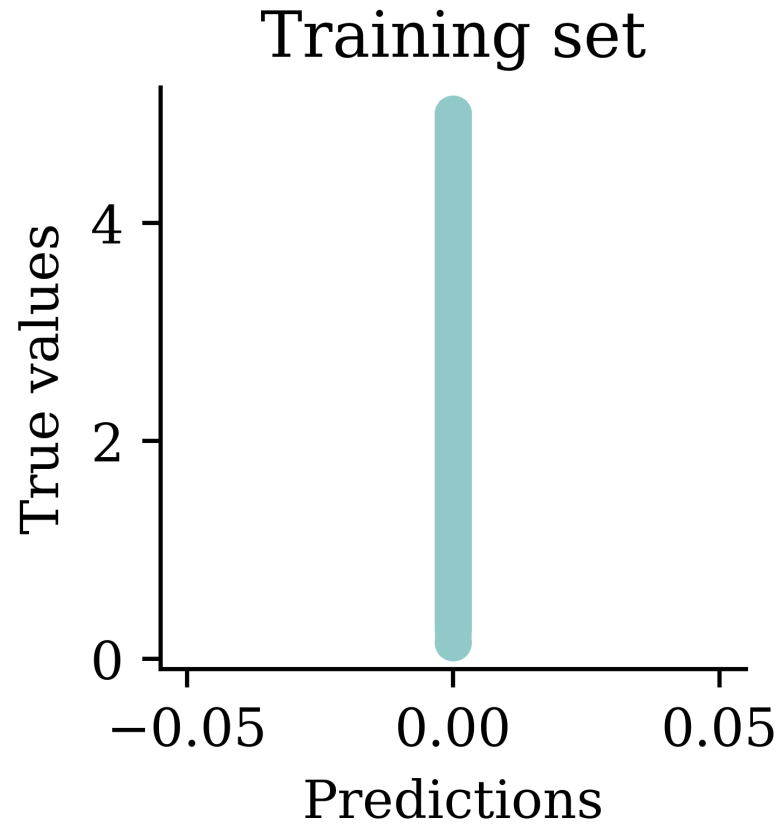
CPU times: user 13.4 s, sys: 4.22 ms, total: 13.4 s

Wall time: 13.4 s

[5.65 5.64 5.64 5.64 5.64] ... [5.64 5.64 5.64 5.64 5.64]



Plot the predictions



Enforce positive outputs (e^x)

```
1 random.seed(123)
2
3 model = Sequential([
4     Dense(30, activation="relu"),
5     Dense(1, activation="exponential")
6 ])
7
8 model.compile("adam", "mse")
9
10 %time hist = model.fit(X_train, y_train, epochs=5, verbose=False)
11
12 losses = hist.history["loss"]
13 print(losses)
```

CPU times: user 1.5 s, sys: 793 μ s, total: 1.5 s

Wall time: 1.35 s

[25.20479965209961, 5.639228820800781, 5.653264045715332, 5.639530658721924,
5.6393961906433105]



Lecture Outline

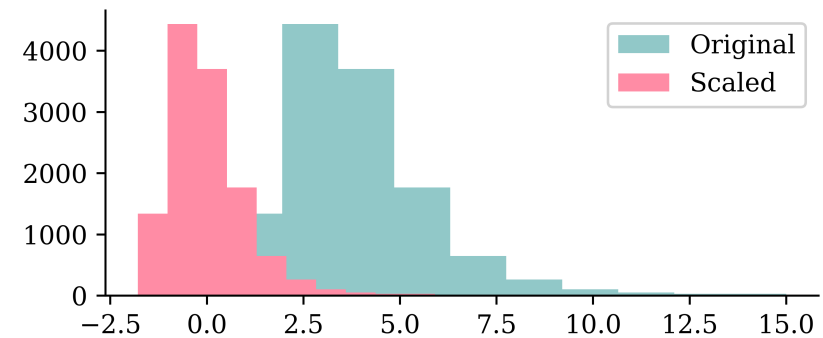
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Re-scaling the inputs

```
1 from sklearn.preprocessing import StandardScaler, MinMaxScaler
2
3 scaler = StandardScaler()
4 scaler.fit(X_train)
5
6 X_train_sc = scaler.transform(X_train)
7 X_val_sc = scaler.transform(X_val)
8 X_test_sc = scaler.transform(X_test)
```

```
1 plt.hist(X_train.iloc[:, 0])
2 plt.hist(X_train_sc[:, 0])
3 plt.legend(["Original", "Scaled"]);
```



Same model with scaled inputs

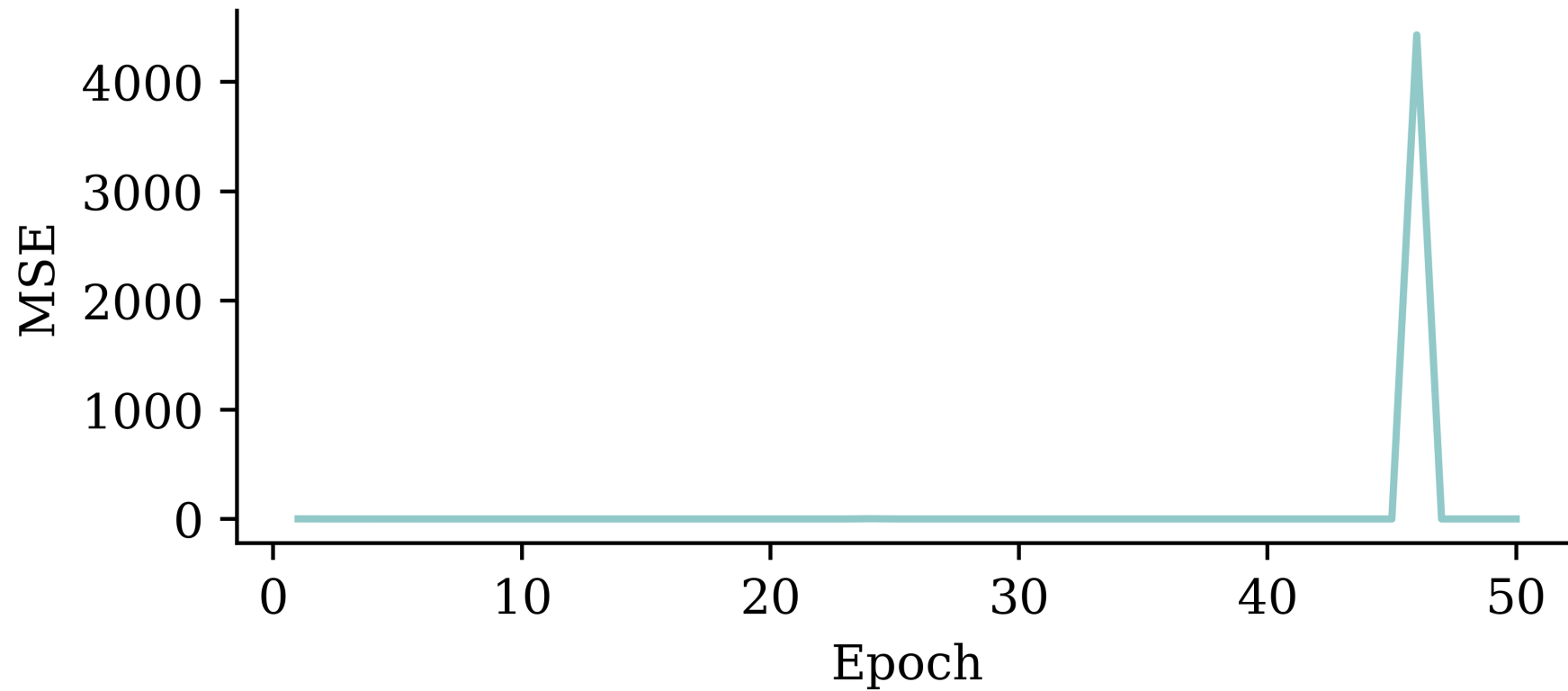
```
1 random.seed(123)
2
3 model = Sequential([
4     Dense(30, activation="relu"),
5     Dense(1, activation="exponential")
6 ])
7
8 model.compile("adam", "mse")
9
10 %time hist = model.fit( \
11     X_train_sc, \
12     y_train, \
13     epochs=50, \
14     verbose=False)
```

CPU times: user 12.9 s, sys: 3.99 ms, total: 12.9 s
Wall time: 12.9 s



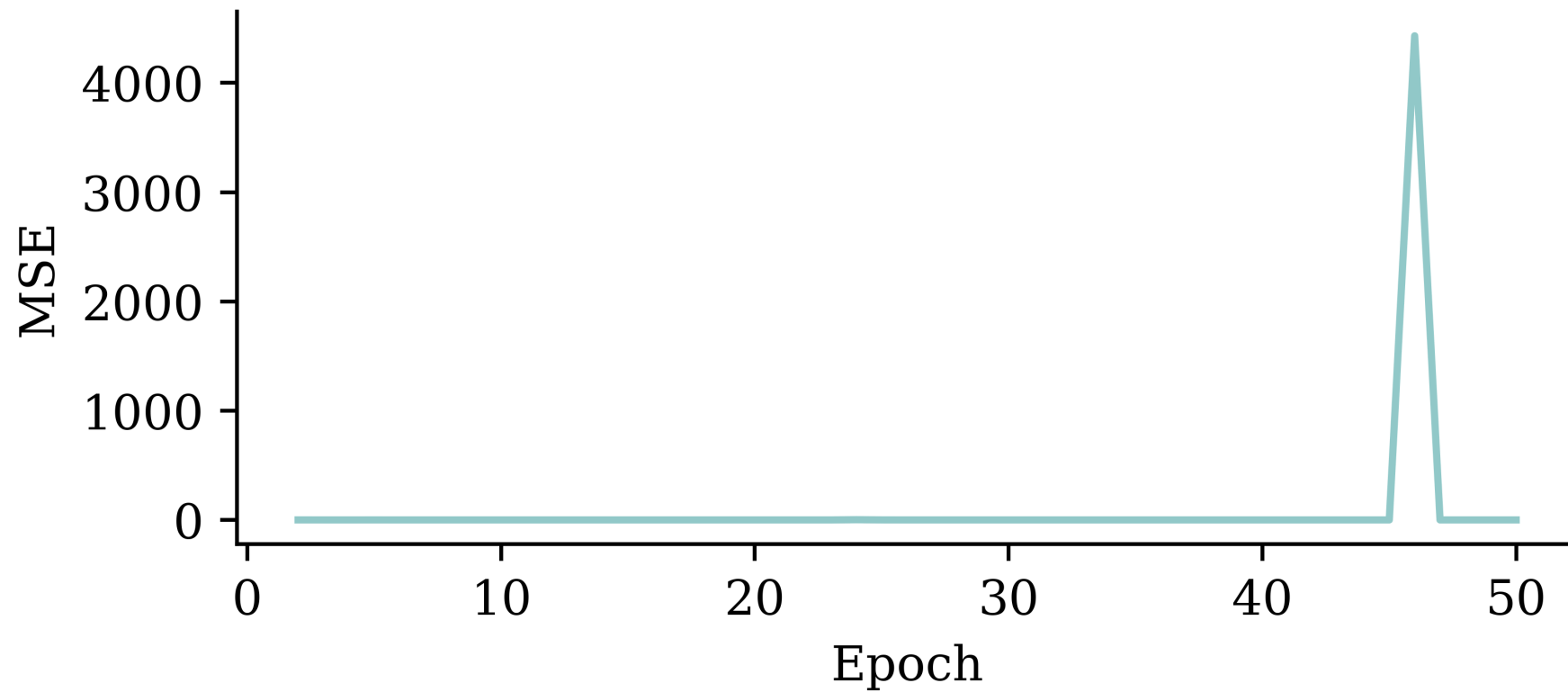
Loss curve

```
1 plt.plot(range(1, 51), hist.history["loss"])
2 plt.xlabel("Epoch")
3 plt.ylabel("MSE");
```



Loss curve

```
1 plt.plot(range(2, 51), hist.history["loss"][1:])  
2 plt.xlabel("Epoch")  
3 plt.ylabel("MSE");
```



Predictions

```

1 y_pred = model.predict(X_val_sc, verbose=0)
2 print(f"Min prediction: {y_pred.min():.2f}")
3 print(f"Max prediction: {y_pred.max():.2f}")

```

Min prediction: 0.00
Max prediction: 8.70

```

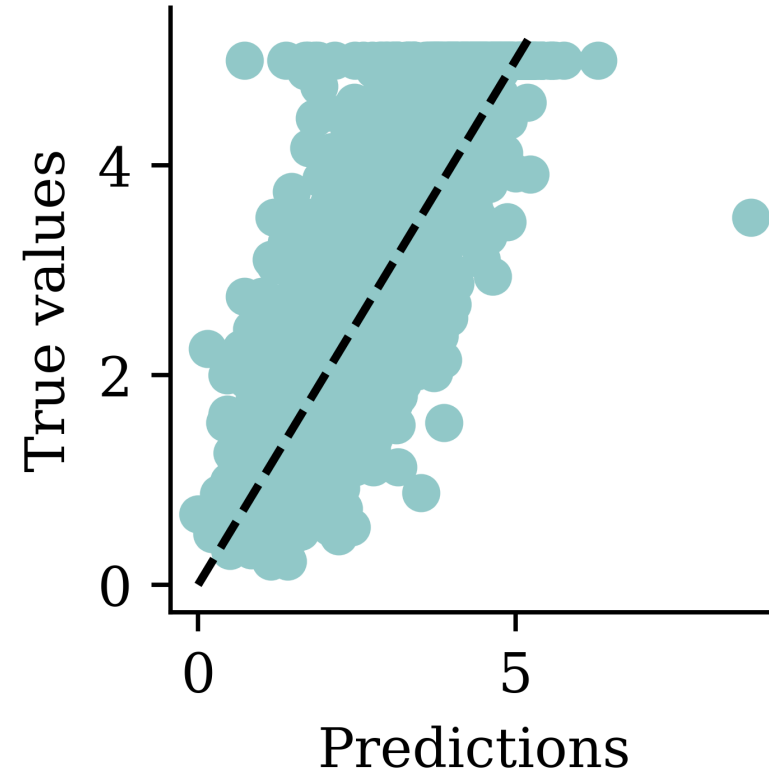
1 plt.scatter(y_pred, y_val)
2 plt.xlabel("Predictions")
3 plt.ylabel("True values")
4 add_diagonal_line()

```

```

1 mse_train["Exp ANN"] = mse(
2     y_train, model.predict(X_train_sc,
3 )
4 mse_val["Exp ANN"] = mse(y_val, model.p

```



Comparing MSE (smaller is better)

On training data:

```
1 mse_train
```

```
{'Linear Regression': 0.5291948207479794,  
'Basic ANN': 1.0832105783628923,  
'Long run ANN': 1.3167506000465417,  
'Exp ANN': 0.32815584170407036}
```

On validation data (expect *worse*, i.e. bigger):

```
1 mse_val
```

```
{'Linear Regression': 0.505942020538137,  
'Basic ANN': 0.8456196916319004,  
'Long run ANN': 1.229804023201492,  
'Exp ANN': 0.3303330511029448}
```



Comparing models (train)

```
1 train_results = pd.DataFrame(  
2     {"Model": mse_train.keys(), "MSE": mse_train.values()}  
3 )  
4 train_results.sort_values("MSE", ascending=False)
```

	Model	MSE
2	Long run ANN	1.316751
1	Basic ANN	1.083211
0	Linear Regression	0.529195
3	Exp ANN	0.328156



Comparing models (validation)

```
1 val_results = pd.DataFrame(  
2     {"Model": mse_val.keys(), "MSE": mse_val.values()}  
3 )  
4 val_results.sort_values("MSE", ascending=False)
```

	Model	MSE
2	Long run ANN	1.229804
1	Basic ANN	0.845620
0	Linear Regression	0.505942
3	Exp ANN	0.330333

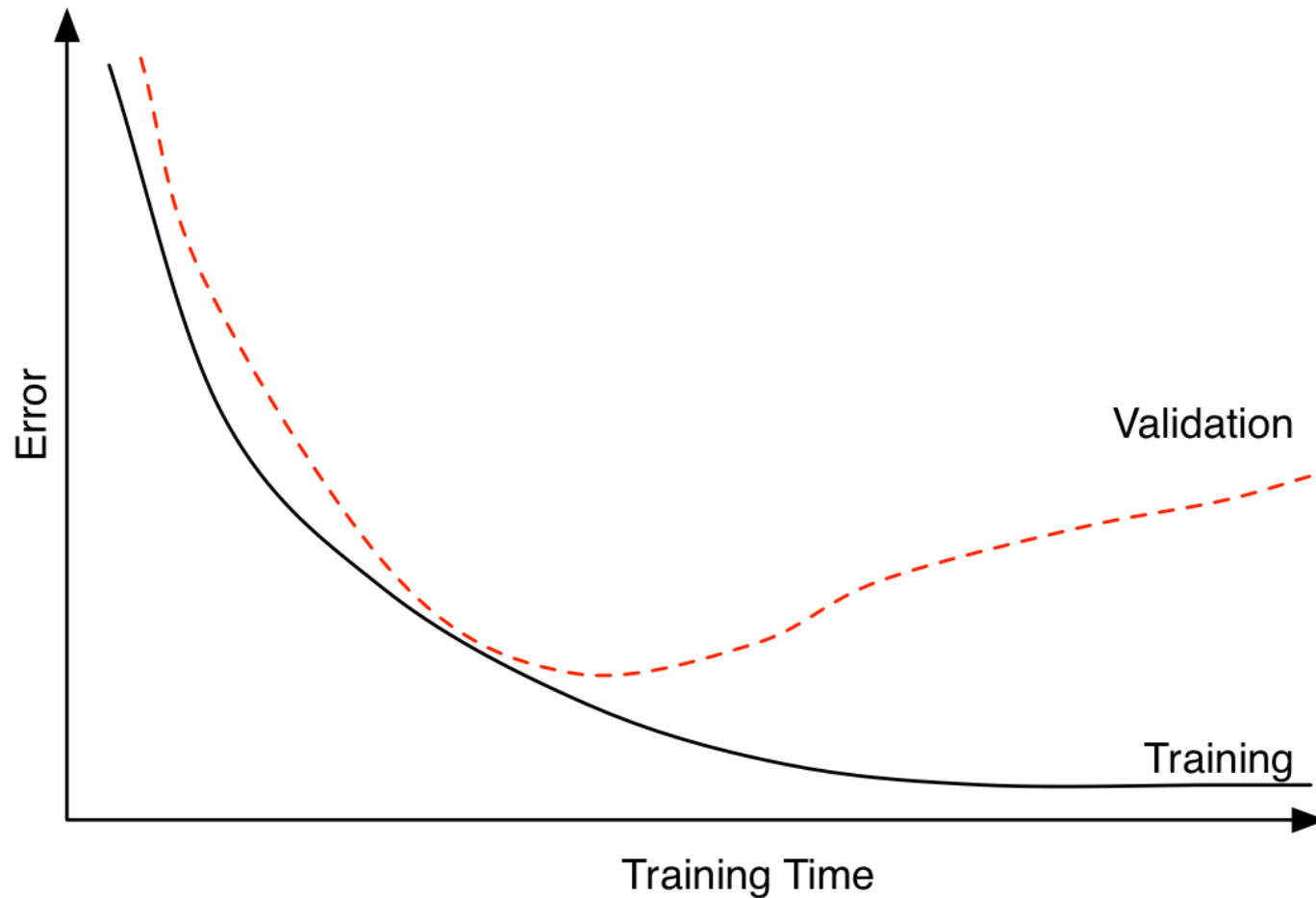


Lecture Outline

- California House Price Prediction
- EDA & Baseline Model
- Our First Neural Network
- Force positive predictions
- Preprocessing
- **Early Stopping**
- Quiz



Choosing when to stop training



Illustrative loss curves over time.

Source: Heaton (2022), [Applications of Deep Learning](#), Part 3.4: Early Stopping.



Try early stopping

Hinton calls it a “beautiful free lunch”

```
1 from keras.callbacks import EarlyStopping
2
3 random.seed(123)
4 model = Sequential([
5     Dense(30, activation="relu"),
6     Dense(1, activation="exponential")
7 ])
8 model.compile("adam", "mse")
9
10 es = EarlyStopping(restore_best_weights=True, patience=15)
11
12 %time hist = model.fit(X_train_sc, y_train, epochs=1_000, \
13     callbacks=[es], validation_data=(X_val_sc, y_val), verbose=False)
14 print(f"Keeping model at epoch #{len(hist.history['loss'])-10}.")
```

CPU times: user 33.8 s, sys: 13.3 ms, total: 33.8 s

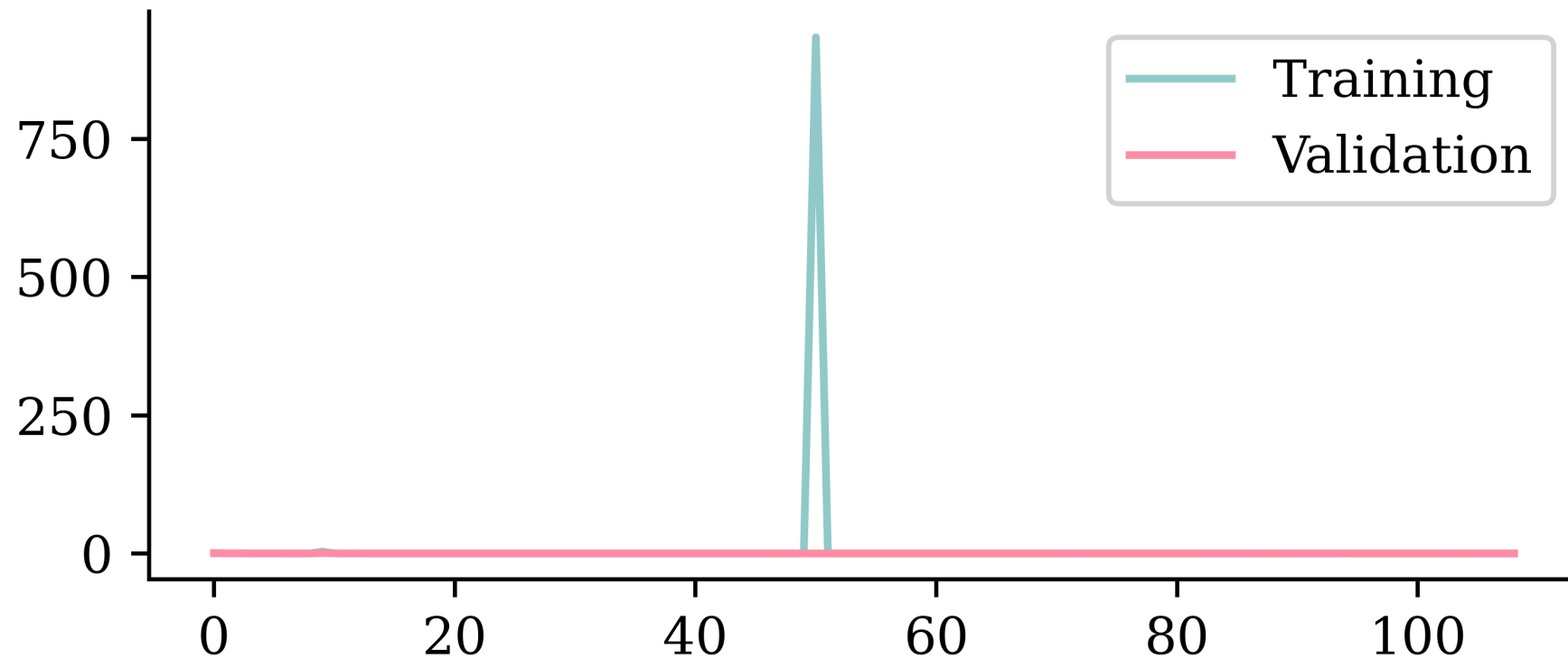
Wall time: 33.7 s

Keeping model at epoch #99.



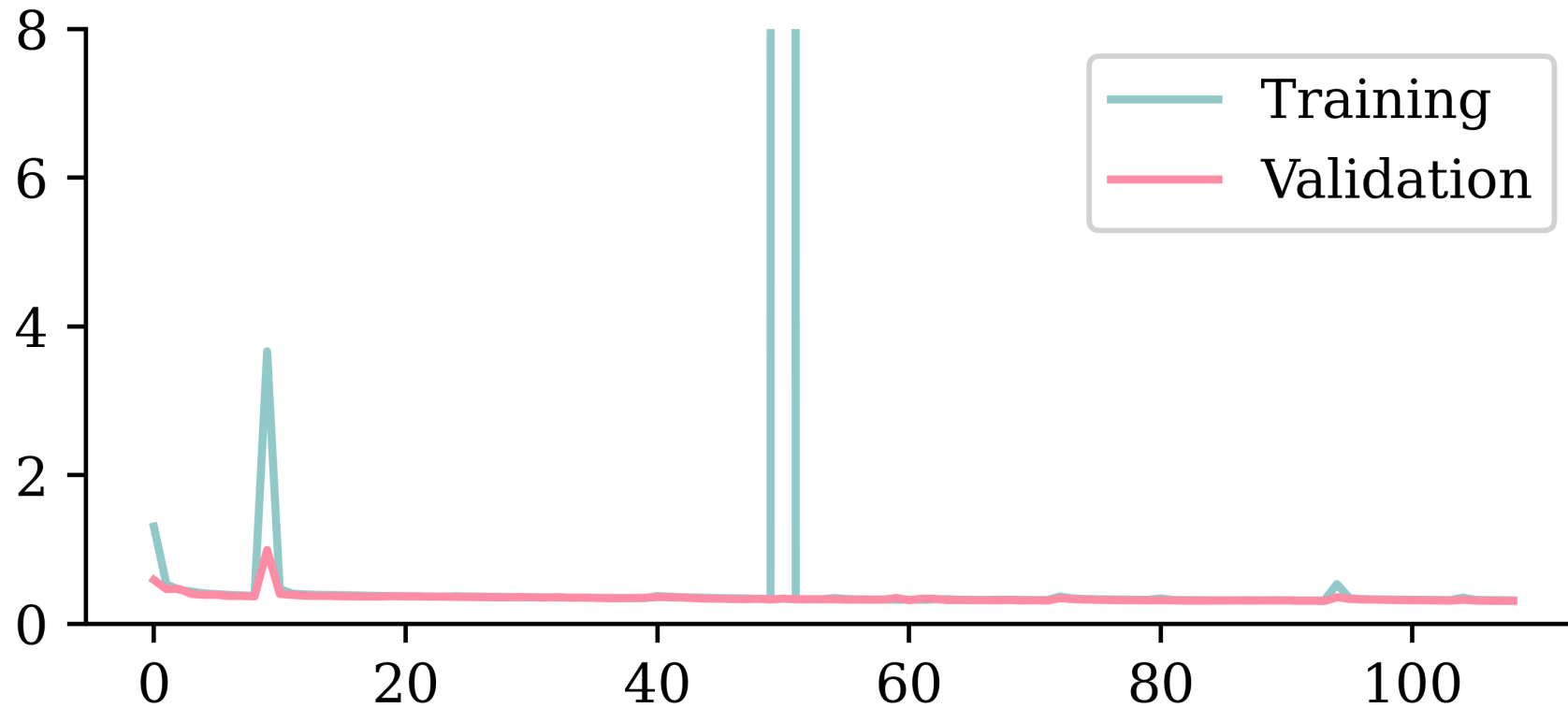
Loss curve

```
1 plt.plot(hist.history["loss"])
2 plt.plot(hist.history["val_loss"])
3 plt.legend(["Training", "Validation"]);
```

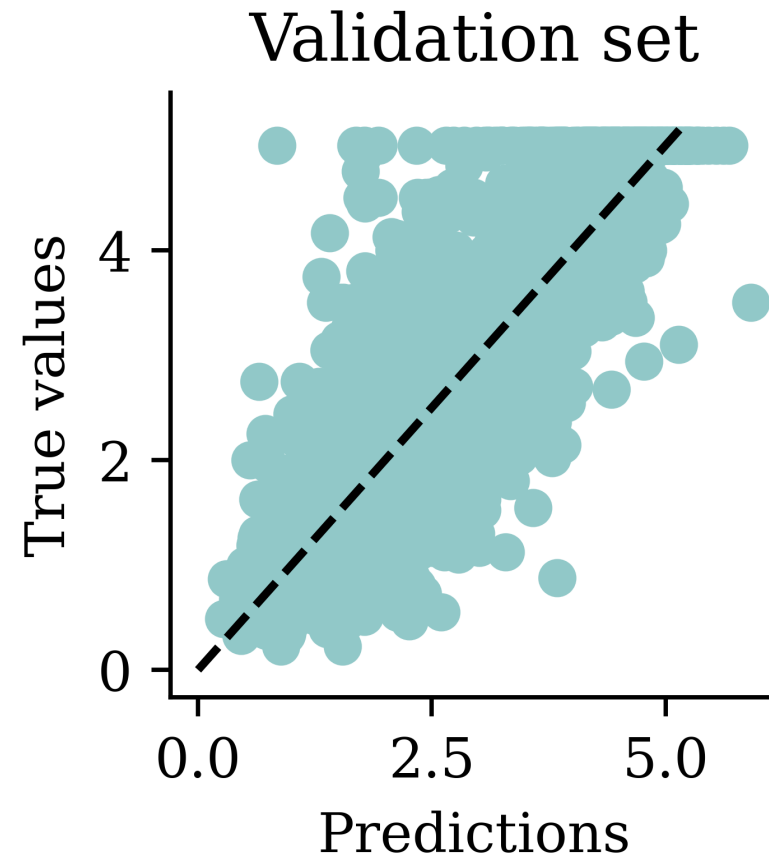
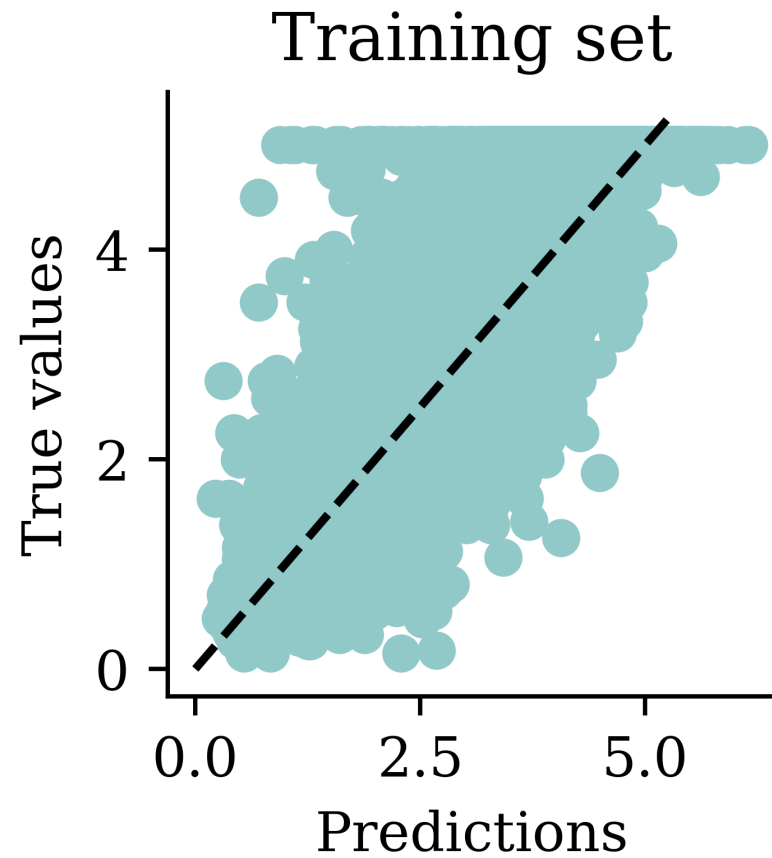


Loss curve II

```
1 plt.plot(hist.history["loss"])
2 plt.plot(hist.history["val_loss"])
3 plt.ylim([0, 8])
4 plt.legend(["Training", "Validation"]);
```



Predictions



Comparing models (validation)

	Model	MSE
2	Long run ANN	1.229804
1	Basic ANN	0.845620
0	Linear Regression	0.505942
3	Exp ANN	0.330333
4	Early stop ANN	0.307353



The test set

Evaluate *only the final/selected model* on the test set.

```
1 mse(y_test, model.predict(X_test_sc, verbose=0))
```

```
0.32820836510257595
```

```
1 model.evaluate(X_test_sc, y_test, verbose=False)
```

```
0.32820841670036316
```



Another useful callback

```
1 from pathlib import Path
2 from keras.callbacks import ModelCheckpoint
3
4 random.seed(123)
5 model = Sequential(
6     [Dense(30, activation="relu"), Dense(1, activation="exponential")]
7 )
8 model.compile("adam", "mse")
9 mc = ModelCheckpoint(
10     "best-model.keras", monitor="val_loss", save_best_only=True
11 )
12 es = EarlyStopping(restore_best_weights=True, patience=5)
13 hist = model.fit(
14     X_train_sc,
15     y_train,
16     epochs=100,
17     validation_split=0.1,
18     callbacks=[mc, es],
19     verbose=False,
20 )
21 Path("best-model.keras").stat().st_size
```

19195



Lecture Outline

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Critique this 🍌 regression code

```
1 X_train = features[:80]; X_test = features[81:]
2 y_train = targets[:80]; y_test = targets[81:]
```

```
1 model = Sequential([
2     Input((2,)),
3     Dense(32, activation='relu'),
4     Dense(32, activation='relu'),
5     Dense(1, activation='sigmoid')
6 ])
7 model.compile(optimizer="adam", loss='mse')
8 es = EarlyStopping(patience=10)
9 fitted_model = model.fit(X_train, y_train, epochs=5,
10     callbacks=[es], verbose=False)
```

```
1 trainMAE = model.evaluate(X_train, y_train, verbose=False)
2 hist = model.fit(X_test, y_test, epochs=5,
3     callbacks=[es], verbose=False)
4 hist.history["loss"]
5 testMAE = model.evaluate(X_test, y_test, verbose=False)
```

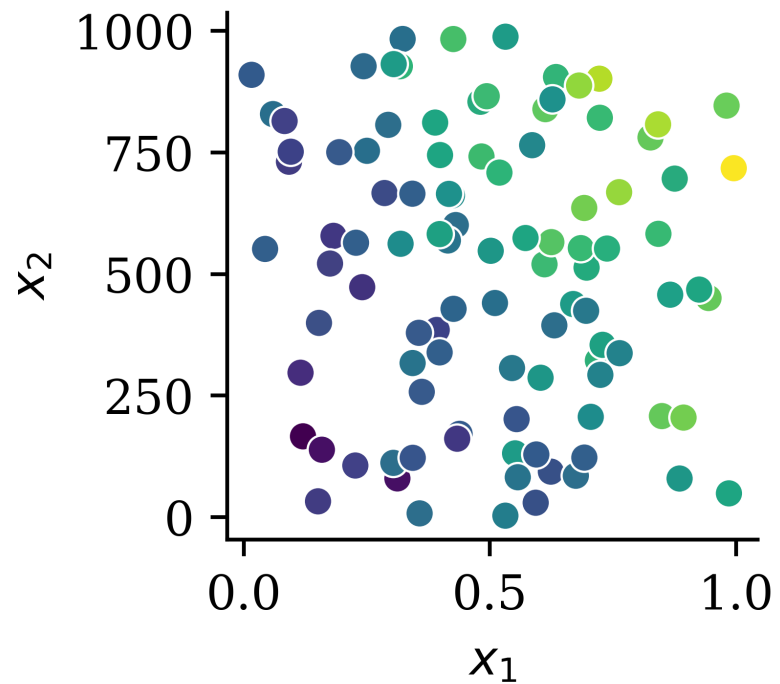
```
1 f"Train MAE: {testMAE:.2f} Test MAE: {trainMAE:.2f}"
```

```
'Train MAE: 4.85 Test MAE: 4.32'
```

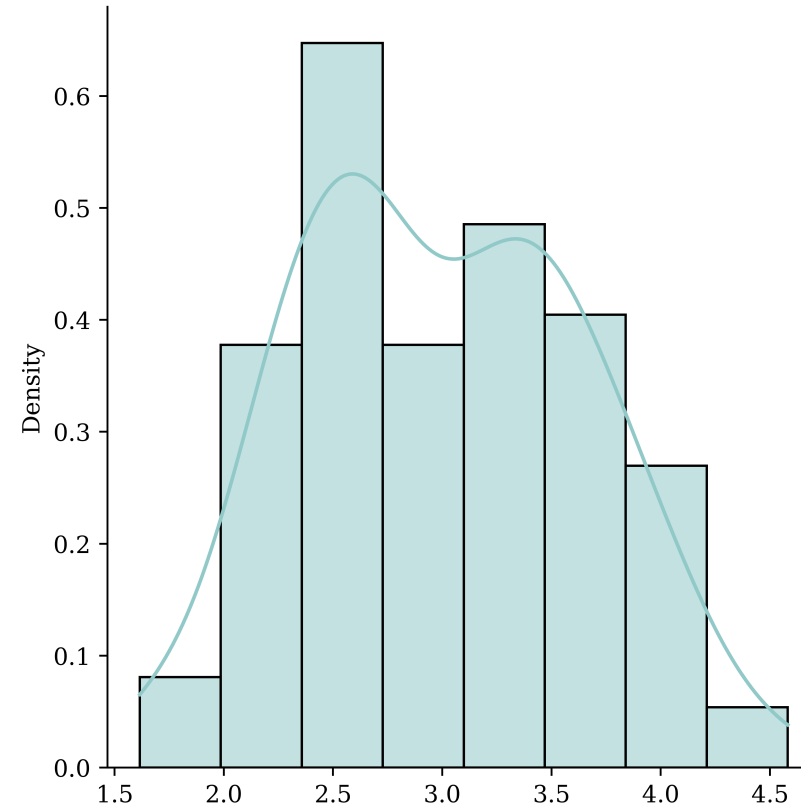


The data

```
1 sns.scatterplot(  
2     x="$x_1$", y="$x_2$",  
3     c=targets, data=features);
```



```
1 sns.displot(targets, kde=True, stat="de
```



Package Versions

```
1 from watermark import watermark
2 print(watermark(python=True, packages="keras,matplotlib,numpy,pandas,seaborn,scipy,torch"))
```

```
Python implementation: CPython
Python version       : 3.11.8
IPython version      : 8.23.0
```

```
keras      : 3.2.0
matplotlib: 3.8.4
numpy      : 1.26.4
pandas     : 2.2.1
seaborn    : 0.13.2
scipy      : 1.11.0
torch      : 2.2.2
tensorflow: 2.16.1
tf_keras   : 2.16.0
```



Glossary

- callbacks
- cost/loss function
- early stopping
- epoch
- Keras, Tensorflow, PyTorch
- matplotlib, seaborn
- neural network architecture
- targets
- training/test split
- validation set

